Name:                                    Date:
Instructor:                              Section:


# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

### 6.1 Multiplying and Simplifying Rational Expressions

| Learning Objectives |
| --- |
| a    Find all numbers for which a rational expression is not defined. |
| b    Multiply a rational expression by 1, using an expression such as *A / A.* |
| c    Simplify rational expressions by factoring the numerator and the denominator and removing factors of 1. |
| d    Multiply rational expressions and simplify. |

### Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

  **equivalent**            **multiply**            **rational**            **simplify**

1.    A quotient, or ratio, of polynomials is a(n) _____ expression.

2.    Expressions that have the same value for all allowable replacements are called

       _____ expressions.

3.    To _____ rational expressions, multiply numerators and multiply

       denominators.

4.    To _____ rational expressions, factor the numerator and the

       denominator and "remove" a factor of 1.

### Objective a    Find all numbers for which a rational expression is not defined.

*Find all numbers for which a rational expression is not defined.*

5.    $\dfrac{-1}{5x}$                                    5. _____

6.    $\dfrac{3}{x+7}$                                   6. _____

**7.** $\dfrac{2}{4a-5}$

**8.** $\dfrac{5x+1}{x^2-x-6}$

**9.** $\dfrac{n^2-n}{n^2-16}$

**10.** $\dfrac{p+7}{20}$

**Objective b    Multiply a rational expression by 1, using an expression such as $A/A$.**

*Multiply. Do not simplify. Note that in each case you are multiplying by 1.*

**11.** $\dfrac{5a^2}{5a^2}\cdot\dfrac{6c^2}{7d^4}$

**12.** $\dfrac{y-6}{y+1}\cdot\dfrac{y-7}{y-7}$

Name:                                    Date:
Instructor:                              Section:

**Objective c   Simplify rational expressions by factoring the numerator and the denominator and removing factors of 1.**

*Simplify.*

13. $\dfrac{4x^5}{20x}$

13. _____

14. $\dfrac{30a^8b^5}{12ab^4}$

14. _____

15. $\dfrac{8m-40}{8m}$

15. _____

16. $\dfrac{4x^2+8x}{12x^3+4x}$

16. _____

17. $\dfrac{t^2-1}{t^2-10t+9}$

17. _____

18. $\dfrac{a^2+a-2}{a^2+2a-3}$

18. _____

**203**

**19.** $\dfrac{x^2 + 25}{x + 5}$

**20.** $\dfrac{9x^2 - 36}{3x^2 - 12}$

**20.** _____

**21.** $\dfrac{2x^2 + 14x + 20}{6x^2 + 12x - 90}$

**21.** _____

**22.** $\dfrac{c^2 - 12c + 36}{c^2 - 36}$

**22.** _____

**23.** $\dfrac{a - 6}{6 - a}$

**23.** _____

**24.** $\dfrac{x^2 - 100}{10 - x}$

**24.** _____

**204**

**Objective d   Multiply rational expressions and simplify.**

*Multiply and simplify.*

**25.** $\dfrac{5x^6}{3x} \cdot \dfrac{9}{2x}$

25. _____

**26.** $\dfrac{4a}{b^3} \cdot \dfrac{3b}{10a^2}$

26. _____

**27.** $\dfrac{x^2+6x+5}{x^2-4x+3} \cdot \dfrac{x-3}{x+5}$

27. _____

**28.** $\dfrac{t^2-25}{3t^3} \cdot \dfrac{t^2-2t}{t^2+7t+10}$

28. _____

**205**

**29.** $\dfrac{6x^3}{5x^2+30x+45} \cdot \dfrac{5x+15}{3x}$

29. _____

**30.** $\dfrac{x^4-16}{x^4-81} \cdot \dfrac{x^2+9}{x^2+4}$

30. _____

**31.** $\dfrac{(m-7)^3}{(m+5)^3} \cdot \dfrac{m^2+10m+25}{m^2-14m+49}$

31. _____

**32.** $\dfrac{3y^2-27}{2y^2-128} \cdot \dfrac{8y+64}{6y-6}$

32. _____

**206**

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

## 6.2 Division and Reciprocals

---

**Learning Objectives**
a    Find the reciprocal of a rational expression.
b    Divide rational expressions and simplify.

---

**Key Terms**
Use the vocabulary terms listed below to complete each statement in Exercises 1–2.

    **interchange**                           **multiply**

**1.**   To find the reciprocal of a rational expression, _____ the numerator and the denominator.

**2.**   To divide by a rational expression, _____ by its reciprocal.

**Objective a**    **Find the reciprocal of a rational expression.**

*Find the reciprocal.*

**3.**   $\dfrac{3}{y}$                                   **3.** _____

**4.**   $x^2 - 9$                              **4.** _____

**5.**   $\dfrac{1}{n+3}$                            **5.** _____

**6.**   $\dfrac{t^2 + 3t + 7}{t^2 - t - 5}$                  **6.** _____

**Objective b    Divide rational expressions and simplify.**

*Divide and simplify.*

**7.**  $\dfrac{3}{2} \div \dfrac{6}{5}$

**8.**  $\dfrac{5}{t} \div \dfrac{15}{t}$

8. _____

**9.**  $\dfrac{x^2}{y^3} \div \dfrac{x^5}{y}$

9. _____

**10.**  $\dfrac{n+3}{n-5} \div \dfrac{n-3}{n+1}$

10. _____

**11.**  $\dfrac{t^2-4}{t^2} \div \dfrac{t+2}{t-2}$

11. _____

**12.**  $\dfrac{y-7}{18} \div \dfrac{y-7}{3}$

12. _____

208

**13.** $\dfrac{4x+16}{21} \div \dfrac{x+4}{14}$

**13.** _____

**14.** $\dfrac{-8+6x}{15} \div \dfrac{9x-12}{5}$

**14.** _____

**15.** $\dfrac{5a+5}{a-3} \div \dfrac{a+1}{a-7}$

**15.** _____

**16.** $\dfrac{x^2-1}{x^2+1} \div \dfrac{x+1}{x-1}$

**16.** _____

**17.** $\dfrac{c^2-d^2}{8c+8d} \div \dfrac{c+d}{10c}$

**17.** _____

**209**

**18.**  $\dfrac{r^2+9r}{r^2+7r+12} \div \dfrac{2r}{r+4}$

18. _____

**19.**  $\dfrac{4x^2-8x-5}{2x^2-11x-21} \div \dfrac{6x^2-13x-5}{4x^2+8x+3}$

19. _____

**20.**  $\dfrac{a^2+3a+2}{a^2+5a+6} \div \dfrac{a^2-2a-3}{a^2+2a-3}$

20. _____

**21.**  $\dfrac{t^2-4}{6t-12} \div \dfrac{3t+9}{2t^2+6t+4}$

21. _____

**22.**  $\dfrac{x^2-x-12}{2x^2+10x} \div \dfrac{x^2+8x+15}{x^2-x}$

22. _____

**210**

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

## 6.3 Least Common Multiples and Denominators

**Learning Objectives**
a    Find the LCM of several numbers by factoring.
b    Add fractions, first finding the LCD.
c    Find the LCM of algebraic expressions by factoring.

**Key Terms**
Use the vocabulary terms listed below to complete each statement in Exercises 1–2.

**least common denominator**            **least common multiple**

1.    The expression $12x^2y^3$ is the _____ of $6xy^2$, $4x^2y$, and $2y^3$.

2.    The expression $12x^2y^3$ is the _____ of $\dfrac{5}{6xy^2}$, $\dfrac{1}{4x^2y}$, and $\dfrac{3x}{2y^3}$.

**Objective a    Find the LCM of several numbers by factoring.**

*Find the LCM.*

3.    24, 30                                    3. _____

4.    8, 15                                     4. _____

5.    6, 15, 20                                 5. _____

6.    10, 50, 120                               6. _____

**Objective b    Add fractions, first finding the LCD.**

*Add, first finding the LCD. Simplify if possible.*

**7.**   $\dfrac{5}{12} + \dfrac{2}{15}$

7. _____

**8.**   $\dfrac{1}{8} + \dfrac{3}{20}$

8. _____

**9.**   $\dfrac{5}{18} + \dfrac{11}{24}$

9. _____

**10.**  $\dfrac{3}{15} + \dfrac{7}{20} + \dfrac{2}{25}$

10. _____

**Objective c    Find the LCM of algebraic expressions by factoring.**

*Find the LCM.*

**11.**  $10x^3,\ 30x^5$

11. _____

**212**

**12.** $3y^3,\ 12x^2y,\ 15xy^4$

12. _____

**13.** $5(t+7),\ 45(t+7)$

13. _____

**14.** $x,\ x+10,\ x-10$

14. _____

**15.** $a^2-9,\ a^2-2a-3$

15. _____

**16.** $m^3+6m^2+9m,\ m^2-3m$

16. _____

**17.** $x+5,\ (x-5)^2,\ x^2-25$

17. _____

**213**

**18.** $x^2 - x - 30,\ x^2 - 7x + 6$  **18.** _____

**19.** $4 + 5x,\ 16 - 25x^2,\ 4 - 5x$  **19.** _____

**20.** $8a^2 - 16a,\ 2a^2 + 6a - 20$  **20.** _____

**21.** $4y^5 - 24y^4 + 36y^3,\ 6y^3 + 12y^2 - 90y$  **21.** _____

**22.** $10x^2 - 10,\ 8x + 8,\ 6x^3 - 12x^2 + 6x$  **22.** _____

**214**

Name:                                    Date:
Instructor:                              Section:

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

**6.4 Adding Rational Expressions**

**Learning Objective**
a    Add rational expressions.

**Key Terms**
Use the vocabulary terms listed below to complete the steps for adding rational expressions with different denominators in Exercises 1–4.

     **equivalent expression**            **least common multiple**

     **numerators**                   **simplify**

**1.**    Find the _____ of the denominators.

**2.**    For each rational expression, find an _____ with the LCD.

**3.**    Add the _____.

**4.**    _____ if possible.

**Objective a    Add rational expressions.**

*Add. Simplify if possible.*

**5.**    $\dfrac{1}{10} + \dfrac{3}{10}$                           **5.** _____

**6.**    $\dfrac{4x}{x+5} + \dfrac{2x-3}{x+5}$               **6.** _____

**7.**  $\dfrac{4}{a} + \dfrac{7}{a^2}$

**7.** _____

**8.**  $\dfrac{6}{25n} + \dfrac{2}{15n}$

**8.** _____

**9.**  $\dfrac{8}{c^2 d} + \dfrac{5}{cd^2}$

**9.** _____

**10.**  $\dfrac{2x+y}{x^2 y} + \dfrac{3x-y}{xy^2}$

**10.** _____

**11.**  $\dfrac{9}{7t} + \dfrac{5}{t+3}$

**11.** _____

**12.**  $\dfrac{3a}{a^2 - 36} + \dfrac{a}{a+6}$

**12.** _____

**216**

Name: 

Date: 

Instructor: 

Section: 

**13.** $\dfrac{5}{y-2}+\dfrac{3}{(y-2)^2}$

13. _____

**14.** $\dfrac{t+5}{t}+\dfrac{t}{t+5}$

14. _____

**15.** $\dfrac{2}{x^2-6x-7}+\dfrac{5}{x^2-2x-3}$

15. _____

**16.** $\dfrac{x+1}{x-4}+\dfrac{x-4}{x+1}$

16. _____

**17.** $\dfrac{3x}{x^2-4}+\dfrac{5x}{x^2+2x}$

17. _____

**18.** $\dfrac{8}{y}+\dfrac{10}{-y}$

18. _____

**217**

**19.** $\dfrac{3x+2}{x-10}+\dfrac{6x}{10-x}$

**19.** _____

**20.** $\dfrac{n^2}{n-1}+\dfrac{1}{1-n}$

**20.** _____

**21.** $\dfrac{x+1}{x-3}+\dfrac{2x-3}{3-x}+\dfrac{4(2x+1)}{x-3}$

**21.** _____

**22.** $\dfrac{t+2}{(t-5)(t-3)}+\dfrac{3(t-1)}{(t-5)(3-t)}+\dfrac{(t-2)(t-1)}{(5-t)(t-3)}$

**22.** _____

**23.** $\dfrac{3-a}{49-a^2}+\dfrac{a+2}{a-7}$

**23.** _____

**24.** $\dfrac{4}{x^2-5x+4}+\dfrac{5}{x^2-16}$

**24.** _____

218

Name: _____     Date: _____
Instructor: _____     Section: _____

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

## 6.5 Subtracting Rational Expressions

**Learning Objectives**
a   Subtract rational expressions.
b   Simplify combined additions and subtractions of rational expressions.

**Key Terms**
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

**denominator**               **least common denominator**

**multiply**                  **numerators**

1.  To subtract rational expressions, they must be written with a common

    _____.

2.  The least common multiple of the denominators is the _____.

3.  To subtract rational expressions when the denominators are the same, subtract the

    _____.

4.  When one denominator is the opposite of the other, we _____ one

    expression by $-1/-1$ to obtain a common denominator.

## Objective a   Subtract rational expressions.

*Subtract. Simplify if possible.*

5.  $\dfrac{10}{x} - \dfrac{4}{x}$

5. _____

6.  $\dfrac{n}{n-10} - \dfrac{10}{n-10}$

6. _____

**7.** $\dfrac{2x+5}{x^2+6x-7} - \dfrac{x-2}{x^2+6x-7}$

7. _____

**8.** $\dfrac{t-3}{8} - \dfrac{t+2}{2}$

8. _____

**9.** $\dfrac{3r+t}{5r^2t} - \dfrac{5r-4t}{rt^2}$

9. _____

**10.** $\dfrac{8}{x+1} - \dfrac{5}{x-1}$

10. _____

**11.** $\dfrac{4}{x^2+x-20} - \dfrac{3}{x^2-25}$

11. _____

**12.** $\dfrac{2a-5}{15a} - \dfrac{7a-1}{10a}$

12. _____

**13.** $\dfrac{3}{x} - \dfrac{9}{-x}$

13. _____

**14.** $\dfrac{5-t}{t-4} - \dfrac{3t-2}{4-t}$

14. _____

**15.** $\dfrac{x-3}{x^2-16} - \dfrac{5-x}{16-x^2}$

15. _____

220

**16.** $\dfrac{3a}{a^2-1}-\dfrac{2}{1-a}$

16. _____

**17.** $\dfrac{5}{4y^2-4y}-\dfrac{3}{4y-4}$

17. _____

**18.** $\dfrac{x}{x^2+4x+3}-\dfrac{1}{x^2-1}$

18. _____

**Objective b    Simplify combined additions and subtractions of rational expressions.**

*Perform the indicated operations and simplify.*

**19.** $\dfrac{2(3t+1)}{t-2}-\dfrac{5(2t-3)}{2-t}+\dfrac{7t+6}{t-2}$

19. _____

**20.** $\dfrac{2a-b}{a-b}+\dfrac{a+2b}{b-a}-\dfrac{2a}{a-b}$

20. _____

**221**

**21.** $\dfrac{8}{3x-1} - \dfrac{4}{1-3x} + \dfrac{2x}{3x-1} + \dfrac{x-5}{1-3x}$

**21.** _____

**22.** $\dfrac{x+5}{x-3} - \dfrac{2-x}{x+3} - \dfrac{4x-18}{9-x^2}$

**22.** _____

**23.** $\dfrac{3x}{1-4x} + \dfrac{2x}{4x+1} - \dfrac{1}{16x^2-1}$

**23.** _____

**24.** $\dfrac{1}{a-b} + \dfrac{2}{a-b} + \dfrac{2a}{a^2-b^2}$

**24.** _____

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

## 6.6 Solving Rational Equations

**Learning Objective**
a    Solve rational equations.

## Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

**check**            **clear**            **LCM**            **rational**

1.  The equation $\dfrac{2}{x} = \dfrac{3}{x+1}$ is an example of a(n) _____ equation.

2.  To solve a rational equation, first _____ the equation of fractions.

3.  When solving a rational equation, multiply on both sides of the equation by the

    _____ of all the denominators.

4.  After solving a rational equation, always _____ possible solutions in

    the original equation.

## Objective a    Solve rational equations.

*Solve. Don't forget to check!*

5.  $\dfrac{2}{5} - \dfrac{3}{4} = \dfrac{x}{8}$

5.  _____

**223**

**6.** $\dfrac{2}{3} + \dfrac{3}{4} = \dfrac{1}{x}$

6. _____

**7.** $\dfrac{5}{6} + \dfrac{2}{5} = \dfrac{x}{15}$

7. _____

**8.** $\dfrac{1}{x} = \dfrac{5}{8} - \dfrac{7}{9}$

8. _____

**9.** $\dfrac{1}{5} + \dfrac{1}{8} = \dfrac{1}{t}$

9. _____

**10.** $x + \dfrac{3}{x} = 4$

10. _____

**224**

**11.** $\dfrac{x}{7} - \dfrac{7}{x} = 0$

**11.** _____

**12.** $\dfrac{8}{y} = \dfrac{9}{y} - \dfrac{1}{12}$

**12.** _____

**13.** $\dfrac{6}{5a} + \dfrac{2}{a} = 1$

**13.** _____

**14.** $\dfrac{n-4}{n+1} = \dfrac{6}{11}$

**14.** _____

**15.** $\dfrac{3}{x-1} = \dfrac{2}{x+2}$

**15.** _____

**225**

**16.** $\dfrac{x}{4} - \dfrac{x}{6} = \dfrac{1}{4}$

**16.** _____

**17.** $\dfrac{a+3}{8} - \dfrac{a-3}{6} = 1$

**17.** _____

**18.** $\dfrac{7}{a+3} = \dfrac{4}{a-4}$

**18.** _____

**19.** $\dfrac{x-6}{2x+3} = \dfrac{1}{4}$

**19.** _____

**20.** $\dfrac{x-3}{x-8} = \dfrac{5}{x-8}$

**20.** _____

**226**

Name:                                    Date:
Instructor:                              Section:

**21.**  $\dfrac{3}{x+5} = \dfrac{7}{x}$                                    **21.** _____

**22.**  $\dfrac{x+3}{x+9} = \dfrac{x-2}{x+1}$                              **22.** _____

**23.**  $\dfrac{1}{x+1} + \dfrac{1}{x-1} = \dfrac{1}{x^2-1}$               **23.** _____

**24.**  $\dfrac{3}{a+4} - \dfrac{10}{a^2-16} = 1$                          **24.** _____

**25.**  $\dfrac{5-x}{7-x} = \dfrac{2}{x-7}$                                **25.** _____

**227**

**26.** $2 - \dfrac{x-3}{x+5} = \dfrac{x^2-17}{x+5}$

**26.** _____

**27.** $3x - 1 = \dfrac{4x}{x+2}$

**27.** _____

**28.** $\dfrac{4}{x-3} = \dfrac{2x}{x^2-9} - \dfrac{7}{x+3}$

**28.** _____

**228**

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

## 6.7 Applications Using Rational Equations and Proportions

| Learning Objectives |
| --- |
| a    Solve applied problems using rational equations. |
| b    Solve proportion problems. |

### Key Terms
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

> **proportion**            **ratio**            **rate**            **similar**

1.   A(n) _____ of two quantities is their quotient.

2.   The ratio of two different kinds of measure is called a(n) _____.

3.   The equation $\dfrac{A}{B} = \dfrac{C}{D}$ is an example of a(n) _____.

4.   Two triangles are _____ if their corresponding angles have the same measure.

### Objective a    Solve applied problems using rational equations.

*Solve.*

5.   It takes Alexis 3 hr to clean her family's garage. Alyssa takes 4 hr to do the same job. How long would it take them, working together, to clean the garage?

5. _____

6.   Ethan can weed the flowerbeds by his office in 50 min. Anthony can do the same job in 45 min. How long would it take Ethan and Anthony to weed the beds if they worked together?

6. _____

**229**

**7.** Abigail can file a week's worth of invoices in 75 min. Ava can do the same job in 90 min. How long would it take if they worked together?

7. _____

**8.** Around Town Services owns two copy machines, an HP Color LaserJet 2800 and an HP Color LaserJet CM1015. The 2800 can copy a month's worth of advertising fliers in 5 hr. The CM1015 can do the same job in 12 hr. How long would the copiers take to copy the fliers if they work together?

8. _____

**9.** Maya drives 20 km/h faster than Tara. While Tara travels 180 km, Maya travels 260 km. Find their speeds.

9. _____

**10.** The speed of a freight train is 15 mph slower than the speed of a passenger train. The freight train travels 180 mi in the same time that it takes a passenger train to travel 240 mi. Find the speed of each train.

10. _____

**11.** The speed of Tom's scooter is 16 mph less than the speed of Mary Lynn's motorcycle. The motorcycle can travel 290 mi in the same time that the scooter can travel 210 mi. Find of the speed of each vehicle.

11. _____

**12.** Brody rides his bicycle at the same speed that Aeron rides      **12.** _____

his. It takes Brody $\dfrac{1}{3}$ hr more than it takes Aeron to ride to

work. If Brody is 25 mi from work and Aeron is 20 mi
from work, how long does it take Aeron to ride to work?

**Objective b    Solve proportion problems.**

*Find the ratio of the following. Simplify if possible.*

**13.** 360 mi, 15 gal                                          **13.** _____

**14.** 340 km, 4 hr                                            **14.** _____

*Solve.*

**15.** Approximately 100 cocoa beans are required to make      **15.** _____

$\dfrac{1}{4}$ lb of chocolate. How many beans are required to make

$2\dfrac{1}{2}$ lb of chocolate?

**16.** Teri wrote 72 pages for her novel over a period of 12 days. At this rate, how many pages would she write in 16 days?

**16.** _____

**17.** Linda walked 610 steps in 5 min on an elliptical trainer. At this rate, how many steps would she walk in 12 min?

**17.** _____

**18.** The ratio of buttermilk to whole wheat flour in a flat bread recipe is $\dfrac{2}{3}$. If 3 cups of buttermilk are used, how many cups of whole wheat flour are used?

**18.** _____

**19.** To determine the number of trout in his pond, Oak catches 25 trout, tags them, and lets them loose. Later, he catches 18 trout; 10 of them have tags. Estimate the number of trout in the pond.

**19.** _____

**20.** A sample of 48 memory cards contained 3 defective cards.   **20.** _____
How many defective cards would you expect in a sample
of 192 cards?

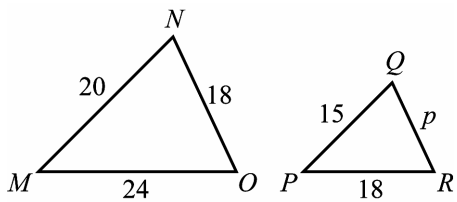*For each pair of similar triangles, find the length of the indicated side.*
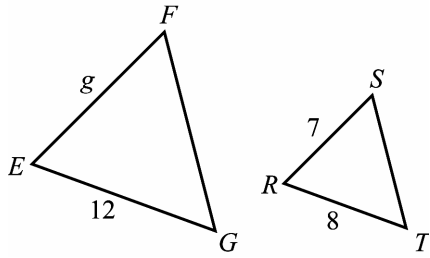
**21.** $x$                                                                   **21.** _____
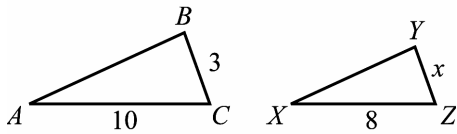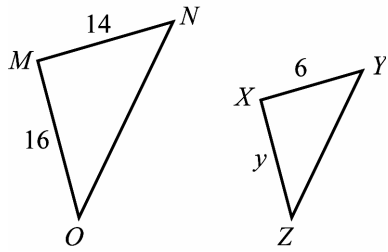


**22.** $p$                                                                   **22.** _____

**23.** *g*



**23.** _____

**24.** *y*



**24.** _____

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

### 6.8 Complex Rational Expressions

| **Learning Objective** |
| --- |
| a    Simplify complex rational expressions. |

**Objective a    Simplify complex rational expressions.**

*Simplify.*

**1.**  $\dfrac{1+\dfrac{7}{12}}{1-\dfrac{2}{3}}$

1. _____

**2.**  $\dfrac{1-\dfrac{3}{8}}{1+\dfrac{7}{8}}$

2. _____

**3.**  $\dfrac{\dfrac{1}{4}+\dfrac{2}{5}}{\dfrac{3}{10}-\dfrac{4}{5}}$

3. _____

**235**

**4.** $\dfrac{\dfrac{1}{a}+2}{\dfrac{1}{a}-4}$

**4.** _____

**5.** $\dfrac{9-\dfrac{1}{t^2}}{3-\dfrac{1}{t}}$

**5.** _____

**6.** $\dfrac{6+\dfrac{6}{n}}{2+\dfrac{2}{n}}$

**6.** _____

**7.** $\dfrac{\dfrac{x}{10}-\dfrac{10}{x}}{\dfrac{1}{x}+\dfrac{1}{10}}$

**7.** _____

**8.** $\dfrac{\dfrac{1}{p}+1}{\dfrac{1}{p^2}-1}$

**8.** _____

236

Name:                      Date:

Instructor:             Section:

**9.** $\quad \dfrac{\dfrac{1}{2}-\dfrac{1}{x}}{\dfrac{2-x}{2}}$

**10.** $\quad \dfrac{\dfrac{3}{y^2}-\dfrac{3}{z^2}}{\dfrac{1}{y}+\dfrac{1}{z}}$

**11.** $\quad \dfrac{x-4-\dfrac{5}{x}}{x-2-\dfrac{3}{x}}$

**12.** $\quad \dfrac{\dfrac{7}{a^3}-\dfrac{2}{a^2}}{\dfrac{5}{a^2}+\dfrac{4}{a}}$

**237**

**13.** $\dfrac{\dfrac{3}{5n^5} - \dfrac{1}{15n}}{\dfrac{6}{7n^3} + \dfrac{2}{21n}}$

**14.** $\dfrac{\dfrac{x}{y} + \dfrac{w}{z}}{\dfrac{y}{x} + \dfrac{z}{w}}$

**15.** $\dfrac{\dfrac{a}{4b^2} + \dfrac{5}{12b}}{\dfrac{5}{12b} + \dfrac{a}{4b^2}}$

**16.** $\dfrac{\dfrac{5}{n+1} + \dfrac{2}{n}}{\dfrac{3}{n+1} + \dfrac{5}{n}}$

238

# Chapter 6 RATIONAL EXPRESSIONS AND EQUATIONS

## 6.9 Direct Variation and Inverse Variation

**Learning Objectives**
a    Find an equation of direct variation given a pair of values of the variables.
b    Solve applied problems involving direct variation.
c    Find an equation of inverse variation given a pair of values of the variables.
d    Solve applied problems involving inverse variation.

**Key Terms**
Use the vocabulary terms listed below to complete each statement in Exercises 1–4.

**direct**                    **inverse**                    **proportionality**     **variation**

1.    The equation $y = kx$ is called an equation of _____ variation.

2.    In the equation $y = kx$, $k$ is called the constant of _____.

3.    The equation $y = k / x$ is called an equation of _____ variation.

4.    In the equation $y = k / x$, $k$ is called the _____ constant.

**Objective a    Find an equation of direct variation given a pair of values of the variables.**

*Find an equation of variation in which $y$ varies directly as x and the following are true. Then find the value of y when $x = 30$.*

5.    $y = 42$ when $x = 6$                    5. _____

6.    $y = 0.3$ when $x = 0.4$                    6. _____

7.    $y = 30$ when $x = 25$                    7. _____

8.    $y = 30$ when $x = 200$                    8. _____

**239**

**Objective b    Solve applied problems involving direct variation.**

*Solve.*

9.  The number of teaspoons $t$ of tea leaves varies directly as the number of cups $C$ of tea made. Lisa uses 15 teaspoons of tea leaves to make 10 cups of tea.
    a)  Find an equation of variation.

    **9. a)** _____

    **b)** _____

    b)  How many teaspoons of tea leaves are needed to make 8 cups of tea?

10. The number of servings $P$ of dried pineapple varies directly as the size $C$ of the container. A 14-oz bag of pineapple contains 10 servings.
    a)  Find an equation of variation.

    **10. a)** _____

    **b)** _____

    b)  How many servings of dried pineapple are contained in a 20-oz carton?

11. The number of calories $c$ burned by a person in a Zumba aerobic class is directly proportional to the time $t$ spent exercising. It takes 10 min to burn 80 calories (*Source: Family Fun and Fitness*). How long would it take to burn 200 calories in the class?

    **11.** _____

240

**12.** The electrical current *I,* in amperes, in a circuit varies      **12.** _____
directly as the voltage *V.* When 12 volts are applied, the
current is 3 amperes. What is the current when 16 volts are
applied?

**Objective c    Find an equation of inverse variation given a pair of values of the variables.**

*Find an equation of variation in which y varies inversely as x and the following are true. Then find the value of x when $x = 20$.*

**13.** $y = 4$ when $x = 15$                   **13.** _____

**14.** $y = 3.5$ when $x = 0.4$               **14.** _____

**15.** $y = 30$ when $x = 12$                **15.** _____

**16.** $y = 0.5$ when $x = 0.8$              **16.** _____

**241**

**Objective d    Solve applied problems involving inverse variation.**

*Solve.*

17. The number of gallons *N* Ash uses to drive to work is inversely proportional to the miles-per-gallon rating *P* of the vehicle he drives. When he drives his Chevrolet Suburban, rated at 16 mpg, he uses 2.5 gal of gas.
    a) Find an equation of variation.

    17. a) _____

        b) _____

    b) How much gas will he use if he rides his Yamaha V-Max, rated at 40 mpg?

18. The time *T* required to do a job varies inversely as the number of people *P* working. It takes 4 hr for 9 people to weed the community garden. How long would it take 10 people to complete the job?

    18. _____

19. The current *I* in an electrical conductor varies inversely as the resistance *R* in the conductor. If the current is $\frac{2}{5}$ ampere when the resistance is 120 ohms, what is the current when the resistance is 150 ohms?

    19. _____

20. The wavelength *W* of a radio wave varies inversely as its frequency *F*. A wave with a frequency of 1600 kilohertz has a length of 225 meters. What is the length of a wave with a frequency of 3000 kilohertz?

    20. _____